ASEN 3502 Computational Methods
In-Class Assignment: Constraints and the KKT Conditions

Name: _____ (Please print clearly)

Choose x and y to minimize $f(x, y) = x^2 + y^2$ subject to two requirements: x and y must sum to 2, and x must be at least 1.5.

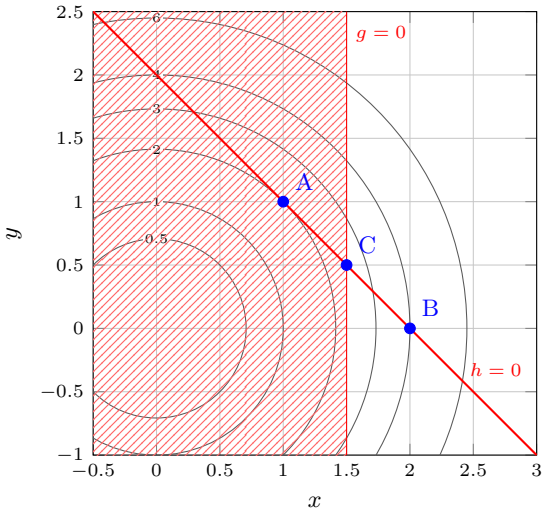
Exercise 1. Write the problem in standard form with $\vec{x} = [x,\ y]^\mathsf{T}$.

$h(\vec{x}) =$ $x + y - 2$ $\quad g(\vec{x}) =$ $1.5 - x$

$x \geq 1.5 \ \Rightarrow\ 0 \geq 1.5 - x \ \Rightarrow\ g(\vec{x}) = 1.5 - x \leq 0$. $h = 2 - x - y$ is also fine (flips sign of λ^*).

Exercise 2. The line $h = 0$ is drawn on the plot. Shade the region where $g > 0$. Which of the points A, B, C are feasible? For each feasible point, which constraints are active?

The feasible set is the part of the line with $x \geq 1.5$. A is infeasible ($g(\text{A}) = 0.5 > 0$). B is feasible with only h active ($g(\text{B}) = -0.5$). C is feasible with both h and g active.



Exercise 3. (a) At each feasible point, compute ∇f, ∇h, and ∇g and solve the *stationarity* condition for candidate multipliers λ^* and σ^*.
(b) Check whether the other KKT conditions hold at each point. Which point satisfies all of them?

(a) B: $\lambda^* =$ 0 $\quad \sigma^* =$ 4 $\quad$ C: $\lambda^* =$ -1 $\quad \sigma^* =$ 2

(b) Point(s) satisfying the KKT conditions: C

(a) $\nabla f = [2x,\ 2y]^\mathsf{T}$, $\nabla h = [1,\ 1]^\mathsf{T}$, $\nabla g = [-1,\ 0]^\mathsf{T}$. Stationarity is $\nabla f + \lambda \nabla h + \sigma \nabla g = \vec{0}$, two equations in two unknowns.

- B $= (2, 0)$: $\begin{bmatrix}4\\0\end{bmatrix} + \lambda \begin{bmatrix}1\\1\end{bmatrix} + \sigma \begin{bmatrix}-1\\0\end{bmatrix} = \vec{0} \ \Rightarrow$ row 2: $\lambda^* = 0$; row 1: $\sigma^* = 4$.
- C $= (1.5, 0.5)$: $\begin{bmatrix}3\\1\end{bmatrix} + \lambda \begin{bmatrix}1\\1\end{bmatrix} + \sigma \begin{bmatrix}-1\\0\end{bmatrix} = \vec{0} \ \Rightarrow$ row 2: $\lambda^* = -1$; row 1: $\sigma^* = 2$.

(b)

	feasible ($h = 0$, $g \leq 0$)	$\sigma^* \geq 0$	$\sigma^* g = 0$
B	yes ($h = 0$, $g = -0.5$)	yes ($\sigma^* = 4$)	**no** ($4 \cdot (-0.5) = -2$)
C	yes ($h = 0$, $g = 0$)	yes ($\sigma^* = 2$)	yes ($2 \cdot 0 = 0$)

B fails *complementary slackness*: g is inactive $g(\vec{x}_B) < 0$, so we must have $\sigma^* = 0$, but stationarity needs $\sigma^* = 4$. C satisfies every condition: $\vec{x}^* = (1.5,\ 0.5)$, $f^* = 2.5$.