Augmented Lagrangian method, one equality constraint $h(\vec{x}) = 0$

input : starting point $\vec{x}_0$, initial multiplier $\lambda_0 = 0$, initial penalty $\mu_0 > 0$, penalty increase factor $\rho > 1$, tolerance E_s

output: $\vec{x}^*$, estimate of the constrained minimizer, and $f(\vec{x}^*)$

`// augmented Lagrangian: Lagrangian plus quadratic penalty`

$$\hat{f}(\vec{x};\, \lambda, \mu) = f(\vec{x}) + \lambda\, h(\vec{x}) + \frac{\mu}{2}\, h(\vec{x})^2$$

$k \leftarrow 0$

while $|h(\vec{x}_k)| > E_s$ **do**

- $\vec{x}_k^* \leftarrow \underset{\vec{x}}{\text{minimize}}\ \hat{f}(\vec{x};\, \lambda_k, \mu_k)$ `// unconstrained solve started from` $\vec{x}_k$
- $\lambda_{k+1} \leftarrow \lambda_k + \mu_k\, h(\vec{x}_k^*)$ `// update multiplier`
- $\mu_{k+1} \leftarrow \rho\, \mu_k$ `// increase penalty`
- $\vec{x}_{k+1} \leftarrow \vec{x}_k^*$ `// next starting point`
- $k \leftarrow k + 1$

end

return $\vec{x}^* = \vec{x}_k,\ \ f(\vec{x}^*)$