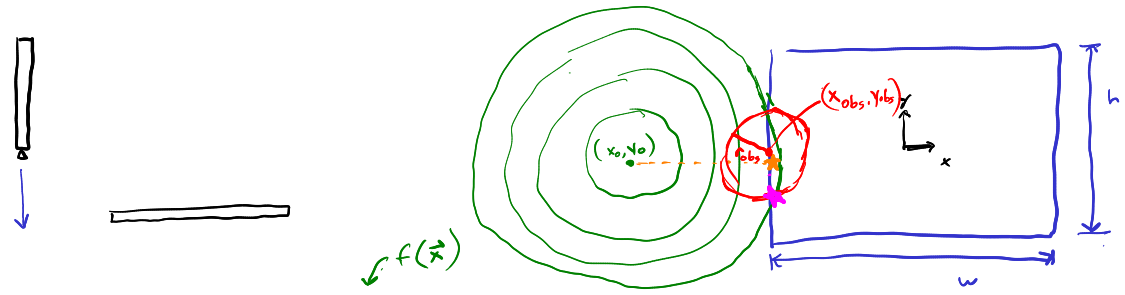
Today

Constrained Optimization
Lagrange Multipliers

Announcements

HW6 Due Thurs
Lab Attendance policy
Vote!



$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\underset{x,y}{\text{minimize}} \quad (x-x_0)^2 + (y-y_0)^2$$

subject to
$$-\frac{w}{2} \le x \le \frac{w}{2}$$
$$-\frac{h}{2} \le y \le \frac{h}{2}$$
$$(x-x_{obs})^2 + (y-y_{obs})^2 \ge r_{obs}^2$$

$$\Rightarrow \quad \vec{g}(\vec{x}) = \begin{bmatrix} -x_1 - \frac{w}{2} \\ x_1 - \frac{w}{2} \\ -x_2 - \frac{h}{2} \\ x_2 - \frac{h}{2} \\ r_{obs}^2 - (x-x_{obs})^2 - (y-y_{obs})^2 \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Standard form

$$\underset{\vec{x}}{\text{minimize}} \quad f(\vec{x})$$

subject to
$$\vec{h}(\vec{x}) = \vec{0}$$
$$\vec{g}(\vec{x}) \le \vec{0}$$

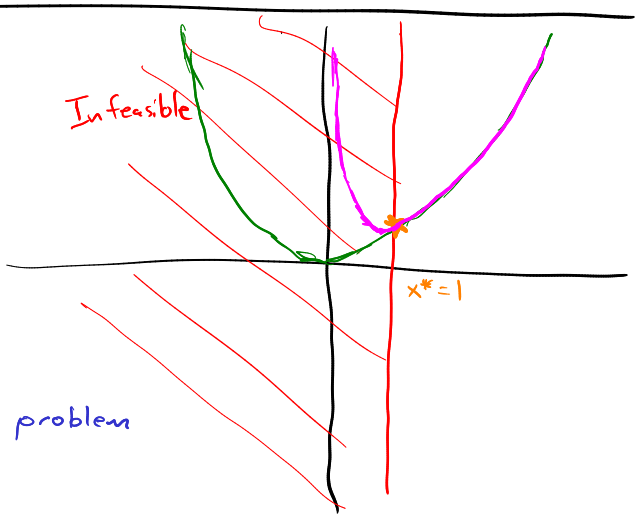
If $\vec{g}(\vec{x}) \le \vec{0}$ and $\vec{h}(\vec{x}) = 0$
$\vec{x}$ is said to feasible

solve with fmincon

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start with scalar

$$\text{minimize} \quad x^2 \qquad f(x) = x^2$$
$$\text{subject to} \quad x \ge 1 \qquad g(x) = 1 - x \le 0$$



Constrained problem ⇒ unconstrained optimization problem

$$\text{minimize} \quad \hat{f}(x)$$

Penalty methods

$$\text{minimize} \quad f(\vec{x}) + \pi(\vec{x}) = \hat{f}(\vec{x})$$
(π: penalty)

Exterior: $$\pi(\vec{x}) = \frac{\mu}{2} \sum_i h_i(\vec{x})^2 + \frac{\mu}{2} \sum_j \max(0, g_j(\vec{x}))^2$$

as μ increases we get closer to true solution, but never becomes feasible

Interior: $$\pi(\vec{x}) = \mu \sum_j -\ln(-g_j(\vec{x}))$$

as μ decreases, gets closer

only works with inequality constraints and feasible initial guess

