Today

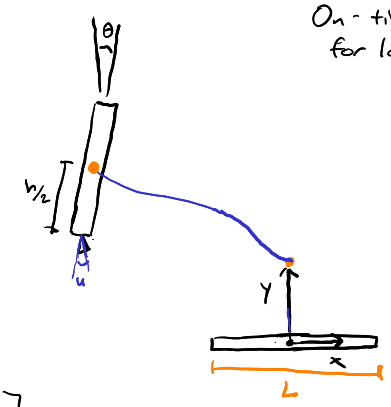
Optimization

Announcements

HW 5 due tomorrow

On-time attendance for lab





$$\vec{x} = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \qquad \dot{\vec{x}} = f(t, \vec{x}, u)$$ ← dynamics

$$\text{minimize } w_1\dot{x}(T)^2 + w_2\dot{y}(T)^2 + w_3\left(y(T) - \frac{h}{2}\right)^2 + w_4\theta(T)^2 + w_5\int_0^T u(t)^2\,dt$$

subject to $-\frac{L}{2} \le x(T) \le \frac{L}{2}$

$u_{min} \le u \le u_{max}$

---

1) Formulation
2) Solution

(often related)

often the hardest

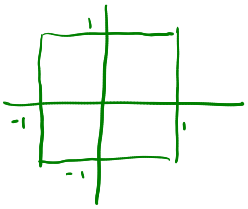
often with an off-the shelf solver fmincon

Unconstrained Optimization Problem

sense → minimize $f(\vec{x})$ ← objective (must return a scalar)

$\vec{x} \in D$ ← domain ← ex. $[-1, 1]$ or $[-1, 1] \times [-1, 1]$

↑ decision variables (can be scalar or vector)



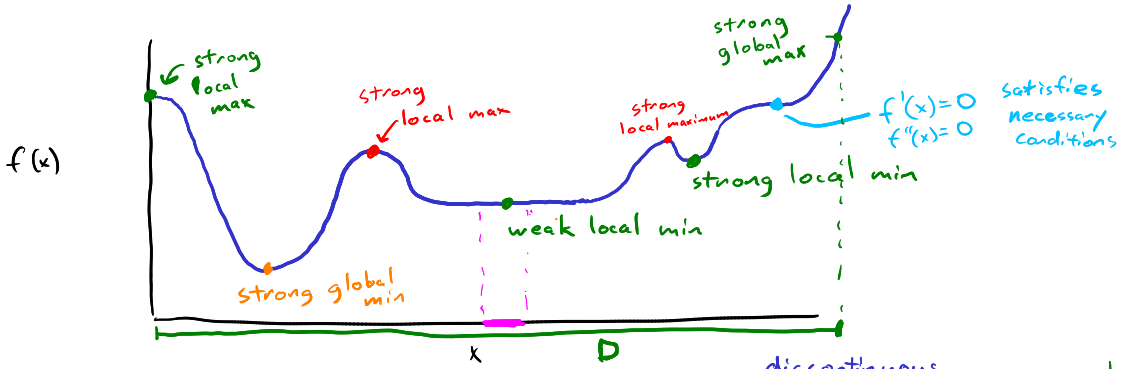
Solutions

- x^* is a global min. if $f(x^*) \le f(x)$ $\forall x \in D$ ← both sides if in interior of D
- x^* is a local min. if $f(x^*) \le f(x)$ $\forall x$ in a neighborhood of x^* within D

x^* is a strong local/global minimum if $f(x^*) < f(x)$ $\forall x \neq x^*$ in a neighborhood of x^* within D

x^* is a weak " " minimum if it is a minimum that is not strong

"extremum" can refer to either a maximum or minimum

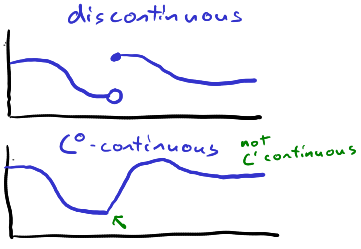


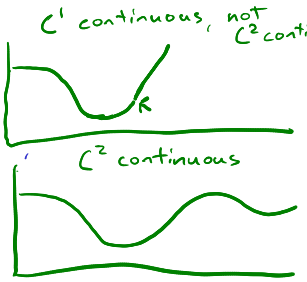
Smoothness

C^0 continuous: f is continuous

C^1 continuous: f' is continuous

C^2 continuous: f'' is continuous





Local Optimality Conditions for C^2 functions

Minimum:

$f'(x^*) = 0$ F.O.N.C. first order necessary condition

$f''(x^*) \geq 0$ S.O.N.C. "stationary points" satisfy this

necessary, but not sufficient

Example of sufficient conditions:

$f'(x^*) = 0$, $f''(x^*) > 0$ ← not true of weak local minimum

x is a scalar

Local optimality conditions for C^2 functions of vectors

F.O.N.C. $\nabla f(\vec{x}^*) = \vec{0}$ $\nabla f(\vec{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$

S.O.N.C. $H(\vec{x}^*)$ is positive semi-definite

"Hessian" $H(\vec{x}) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & & \\ \vdots & & \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$

A symmetric matrix H is positive definite if $\vec{x}^T H \vec{x} > 0 \quad \forall \vec{x} \neq \vec{0}$ or all eigenvalues > 0

A symmetric matrix H is positive semidefinite if $\vec{x}^T H \vec{x} \geq 0 \quad \forall \vec{x} \neq \vec{0}$ or all eigenvalues ≥ 0

Quadratic function

Scalar: $f(x) = \frac{1}{2}ax^2 + bx + c$ $f'(x) = ax + b$ $f''(x) = a$

Vector: $f(\vec{x}) = \frac{1}{2}\vec{x}^T A \vec{x} + \vec{b}^T \vec{x} + c$ $\nabla f(\vec{x}) = A\vec{x} + \vec{b}$ $H = A$

- A: symmetric; controls curvature
- $\vec{b}$: controls where $\vec{x}^*$ is
- c: controls $f(\vec{x}^*)$