

## Brent's Method

**input** : function  $f$ , bracket  $[a, b]$  with  $f(a) \cdot f(b) < 0$ , tolerance  $\varepsilon_s$

**output**: estimate  $b$  of the root

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// b is the best estimate, c is on the other side of the root, a
  is the previous b
c ← a
// d is the signed step b will move by, e is the previous step;
  both start as the bracket width
d ← b - a;   e ← d
repeat
  if f(b) · f(c) > 0 then
    // b and c no longer bracket the root; a does
    c ← a;   d ← b - a;   e ← d
  end
  if |f(c)| < |f(b)| then
    // make b the point with the smallest |f|
    a ← b;   b ← c;   c ← a
  end
  m ← (c - b) / 2                                // bisection step
  δ ← εs |b|                                       // absolute tolerance
  if |m| ≤ δ or f(b) = 0 then
    return b
  end
  if |e| < δ or |f(a)| ≤ |f(b)| then
    d ← m;   e ← m                                // bisect
  else
    s ← f(b) / f(a)
    if a = c then
      // only two distinct points: secant step
      p ← 2ms;   q ← 1 - s
    else
      // three distinct points: inverse quadratic
      interpolation
      q ← f(a) / f(c);   r ← f(b) / f(c)
      p ← s(2mq(q - r) - (b - a)(r - 1))
      q ← (q - 1)(r - 1)(s - 1)
    end
    if p > 0 then q ← -q
    else p ← -p
    if 2p < 3mq - |δq| and p < |eq/2| then
      // interpolated step stays in the bracket and shrinks
      it fast enough
      e ← d;   d ← p / q
    else
      d ← m;   e ← m                                // bisect
    end
  end
end
a ← b
if |d| > δ then b ← b + d
else b ← b + δ sign(m)
until converged
return b
```