# Exam 1 Equation Sheet

(Additions and minor changes may be made by the professor through Thursday, 9/24.)

Absolute: $E$ Approximate: $a$
Relative (percent): $\varepsilon$ True: $t$
Specified: $s$

**function** GAUSS_PIVOT($[A]$, $\{b\}$)
  **for** $k \leftarrow 1$ **to** $n-1$ **do**
    $p \leftarrow \text{argmax}_{i \geq k} |a_{ik}|$
    swap rows $k$ and $p$ of $[A]$ and $\{b\}$
    **for** $i \leftarrow k+1$ **to** $n$ **do**
      $f \leftarrow a_{ik}/a_{kk}$
      **for** $j \leftarrow k$ **to** $n$ **do**
        $a_{ij} \leftarrow a_{ij} - f\, a_{kj}$
      $b_i \leftarrow b_i - f\, b_k$
  $x_n \leftarrow b_n/a_{nn}$
  **for** $i \leftarrow n-1$ **down to** 1 **do**
    $x_i \leftarrow \left(b_i - \sum_{j=i+1}^{n} a_{ij}\, x_j\right)/a_{ii}$
  **return** $\{x\}$

$$\frac{2}{3}n^3 + O(n^2)\text{flops}$$

$$\sum_{i=k}^{m} a\, f(i) = a \sum_{i=k}^{m} f(i)$$

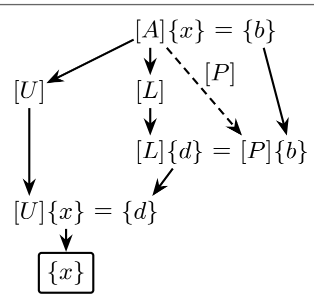
$$\sum_{i=k}^{m} [f(i) + g(i)] = \sum_{i=k}^{m} f(i) + \sum_{i=k}^{m} g(i)$$

$$\sum_{i=1}^{m} 1 = m \qquad \sum_{i=k}^{m} 1 = m - k + 1$$

$$\sum_{i=1}^{m} i = \frac{m(m+1)}{2}$$

$$\sum_{i=1}^{m} i^2 = \frac{m(m+1)(2m+1)}{6}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ f_{21} & 1 & 0 \\ f_{31} & f_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = [P][A]$$




$$\text{cond}[A] = \|[A]\|\,\|[A]^{-1}\|$$

$$\frac{\|\{\Delta x\}\|}{\|\{x\}\|} \leq \text{cond}[A]\,\frac{\|[\Delta A]\|}{\|[A]\|}$$

$$\frac{\|\{\Delta x\}\|}{\|\{x\}\|} \leq \text{cond}[A]\,\varepsilon$$

$$2.22 \times 10^{-16} \qquad 1.19 \times 10^{-7}$$

$E_a = \frac{\Delta x}{n-1}$ $\qquad E_a = \frac{\Delta x}{2^n}$

$n = \frac{\Delta x}{E_s} + 1$ $\qquad n = \log_2\left(\frac{\Delta x}{E_s}\right)$

**function** NEWTON($f$, $f'$, $x_0$)
  $x \leftarrow x_0$
  **for** $k \leftarrow 1$ **to** maxit **do**
    $\Delta x \leftarrow -f(x)/f'(x)$
    $x \leftarrow x + \Delta x$
    **if** $|\Delta x| <$ atol **then return** $x$
  report failure

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$x_{i+1} = x_i - \frac{f(x_i)\,(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$

$\varepsilon_a = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100\%$

$[J](\{x_i\})\,\{\Delta x\} = -\{f\}(\{x_i\})$
$\{x_{i+1}\} = \{x_i\} + \{\Delta x\}$

$E_{a,i+1} = \frac{E_{a,i}}{2}$ $\qquad E_{t,i+1} \approx \frac{-f''(x_r)}{2f'(x_r)} E_{t,i}^2$

$$[J] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Forward: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$ $\qquad h \approx \sqrt{\varepsilon}\, x$

Central: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

$\frac{\partial \{f\}}{\partial x_j} = \frac{\{f\}(\{x\} + h\{e_j\}) - \{f\}(\{x\})}{h}$

No unique $[A]\{x\} = \{b\}$ solution $\iff$ $\det[A] = 0$ $\iff$ $[A]$ is singular $\iff$ $[A]$ not full rank $\iff$ $[A^{-1}]$ does not exist.

$cf(n) = O(f(n)) \quad \forall c > 0$
If $f_1(n) = O(n^a)$ and $f_2(n) = O(n^b)$, then $f_1(n) + f_2(n) = O\left(n^{\max(a,b)}\right)$
If $f_1(n) = O(n^a)$ and $f_2(n) = O(n^b)$, then $f_1(n) \cdot f_2(n) = O\left(n^{a+b}\right)$
If $|f(n)|$ is bounded, $f(n) = O(1)$

`abs(x)` $|x|$ `sign(x)` $-1$, 0, or 1
`norm(x)` $\|x\|_2$ (2-norm of vector or matrix)
`sqrt(x)`, `exp(x)`, `log2(x)`
`[m, i] = max(abs([4,-6,3]))` value & index: `m=6`, `i=2`
`n = length(v); [m, n] = size(A)`
`zeros(m, n)`, `ones(m, n)`, `eye(n)`
`A(i, :)`, `A(:, j)`, `A(i, j:n)`, `&&`, `||`, `~=`, `==`, `~`
`A'` transpose; `A*x`; `x = A\b`
`inv(A)`, `det(A)`, `cond(A)`
`eps`, `Inf`, `NaN`, `isnan(x)`, `isinf(x)`
`@(x) x^2 - 3` defines an anonymous function that computes $x^2 - 3$
`@sin` creates a handle for the function `sin`