ASEN 3502 Computational Methods
Homework 2

See Gradescope for Due Date

Question 1 (LU factorization by hand)
Consider the system $[A]\{x\} = \{b\}$ with

$$[A] = \begin{bmatrix} 3 & 2 \\ 6 & 7 \end{bmatrix}, \qquad \{b\} = \begin{Bmatrix} 7 \\ 20 \end{Bmatrix}.$$

Complete the following exercise by hand, showing your work.

(a) Find a lower triangular matrix, $[L]$, and an upper triangular $[U]$ such that $[A] = [L][U]$.

(b) Now solve $[A]\{x\} = \{b\}$ using forward substitution with $[L]$ and then back substitution with $[U]$.

Question 2 (LU factorization with pivoting by hand)
Consider the same system as Question 1,

$$[A] = \begin{bmatrix} 3 & 2 \\ 6 & 7 \end{bmatrix}, \qquad \{b\} = \begin{Bmatrix} 7 \\ 20 \end{Bmatrix},$$

but now use partial pivoting. Since $|6| > |3|$, the pivot step swaps the two rows.

(a) Find a permutation matrix, $[P]$, such that $[P][A]$ is the new matrix with the rows swapped.

(b) Compute $[P][A]$, and find a lower-triangular matrix, $[L]$, and upper triangular matrix, $[U]$, such that $[P][A] = [L][U]$.

(c) Multiplying both sides of $[A]\{x\} = \{b\}$ by $[P]$ gives $[P][A]\{x\} = [P]\{b\}$, that is, $[L][U]\{x\} = [P]\{b\}$. Compute $[P]\{b\}$, then solve $[L]\{d\} = [P]\{b\}$ by forward substitution.

(d) Solve $[U]\{x\} = \{d\}$ by back substitution. Compare your answer to the one you found in Question 1.

Question 3 (Flop counting and big-O notation)

(a) Let $f(n) = (n-1)^3 + n^2\cos(n) + \frac{\sqrt{n}}{n}$. What is the smallest integer k such that $f(n) = O(n^k)$ as $n \to \infty$. You can use the rules discussed in class instead of using the definition directly.

(b) Assume that the runtime of a numerical solver is dominated by floating point operations. A Gaussian-elimination-based solver uses

$$\frac{2n^3}{3} + \frac{5n^2}{2} - \frac{13n}{6}$$

flops to solve with an $n \times n$ matrix. If a computer takes 0.14 seconds to solve an $n = 1000$ system, how long will it take to solve an $n = 15,000$ system?

(c) Redo the calculation from Part (b) assuming that Gaussian elimination uses n^3 flops (since the true number of flops is $O(n^3)$). By what percentage do the two estimates differ?